Outdoor Ice Rink Environmental Response and Performance Model

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A Mathematical Model to characterize Outdoor Ice Rink Performance
Uses a fully Parameterizable interface and can be expanded for use in other latitudes

This model is derived from the thermodynamical relationships extant in the maintenance of solid ice in outdoor compressor cooled rinks. Energy flux $Q=\frac{dq}{dt}$ at the surface of the ice sheet is calculated once an hour over the months of November to March.

Heat flux at the surface of the ice sheet can be expressed as the sum of the fluxes of the surrounding bodies, namely the three terms in equation (1). Flux at the surface is therefor defined as:

$$Q_{surface}=Q_{solar}+Q_{air}-Q_{cooling\,system} \tag{1}$$

Expanding each term individually gives:

$$Q_{solar}=S \times cloud\,attenuation \times \alpha \sin(\Phi) \tag{2}$$

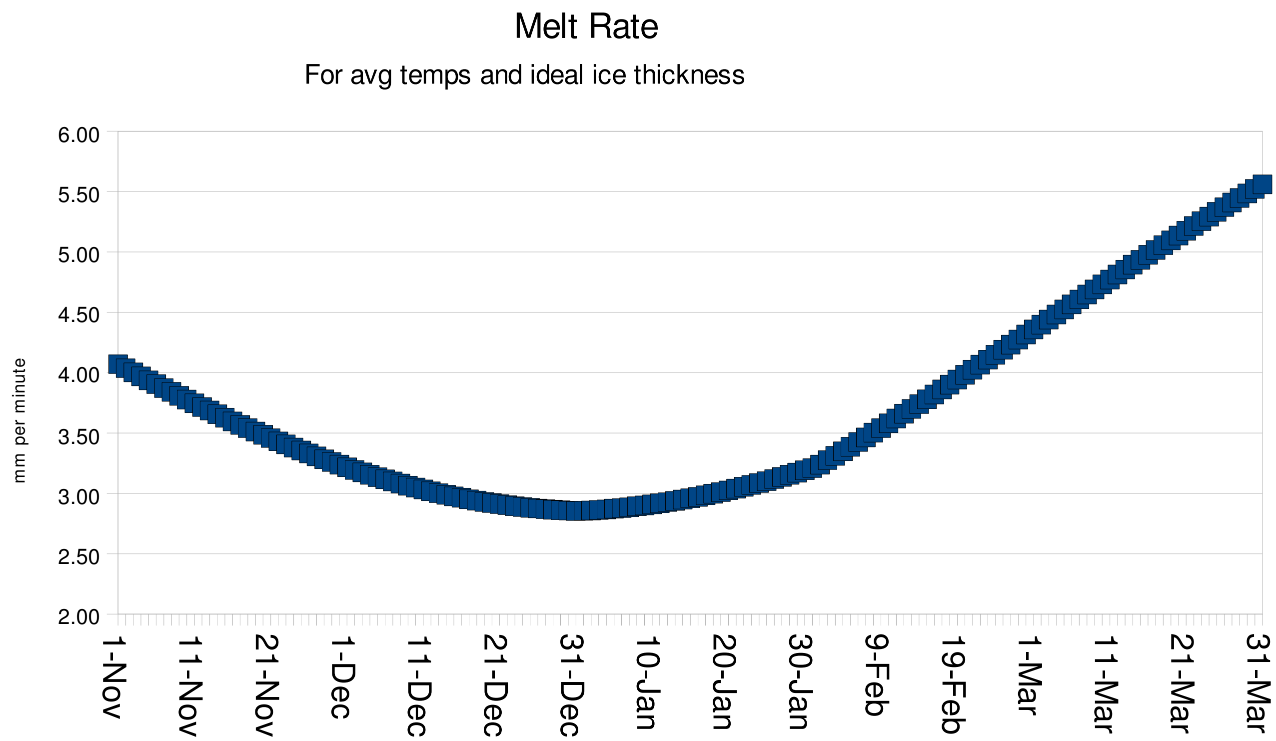
Where S is the solar constant, or the amount of energy emitted by the sun that is incident at the top of the earth's atmosphere normal (at right angle) to the surface. Cloud attenuation is taken to be 0.44 (fair conditions) as this the global average across most of the radiative spectrum. For simulations of overcast conditions, the attenuation is raised to 0.70. Further atmospheric attenuation and surface reflection is accounted for as a function of the incident solar angle to the earth's surface in the $\alpha \sin(\Phi)$ term. Where α is the surface albedo of ice and Φ is the elevation angle of the sun.

$$Q_{air}=h_{air\rightarrow ice}\ A\ \Delta T \tag{3}$$

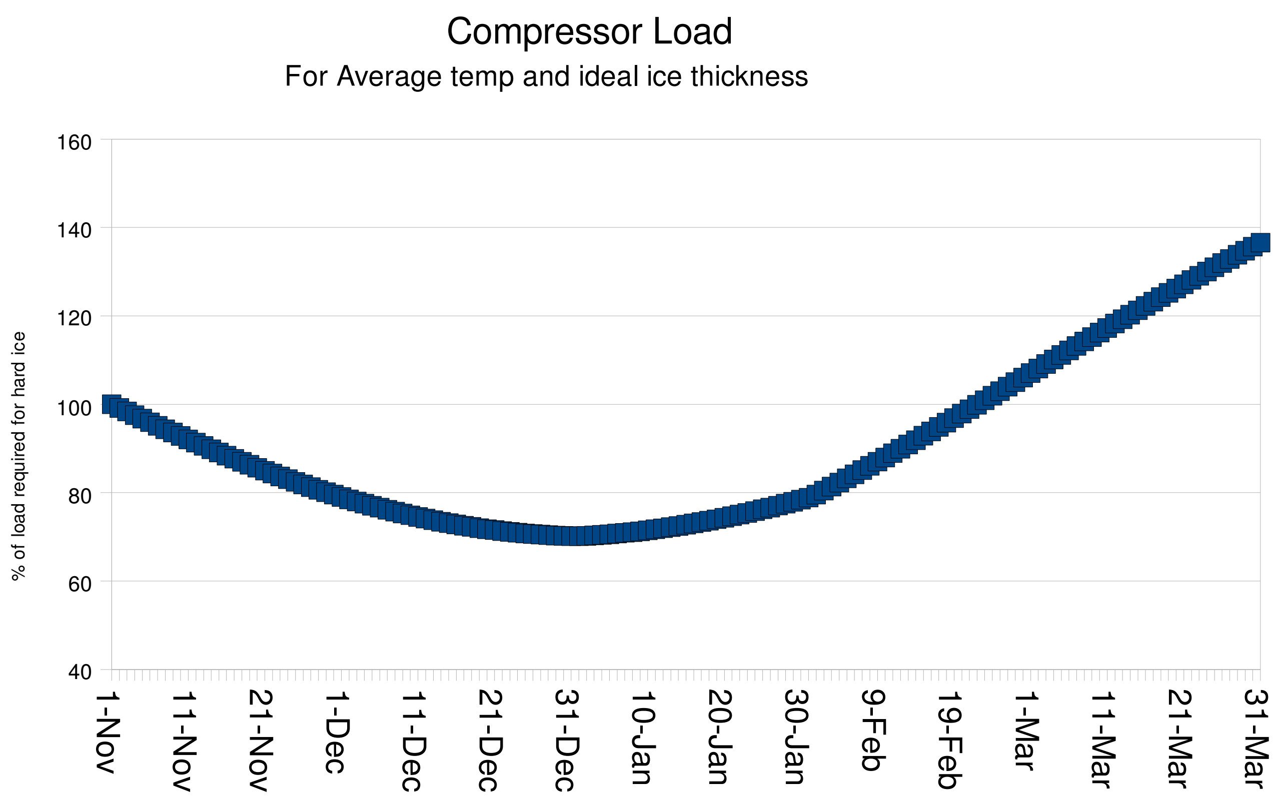
Where $h_{air\rightarrow ice}$ is the heat transfer coefficient between ice and air. A being the surface area and ΔT being the temperature differential between the ice and the ambient air. And:

$$Q_{coolingsystem}=\frac{h_{ice}\ A\ \Delta T}{d} \tag{4}$$

Where h_{ice} is the thermal conductivity of solid ice and d is the thickness of the ice slab. T_2 is taken to be the brine temperature which is easily monitored by plant managers and for the initial run of the model was set at -10∘C. The results of this simulation are then used to compute the melt rate of the surface layer (depth 0.002m).



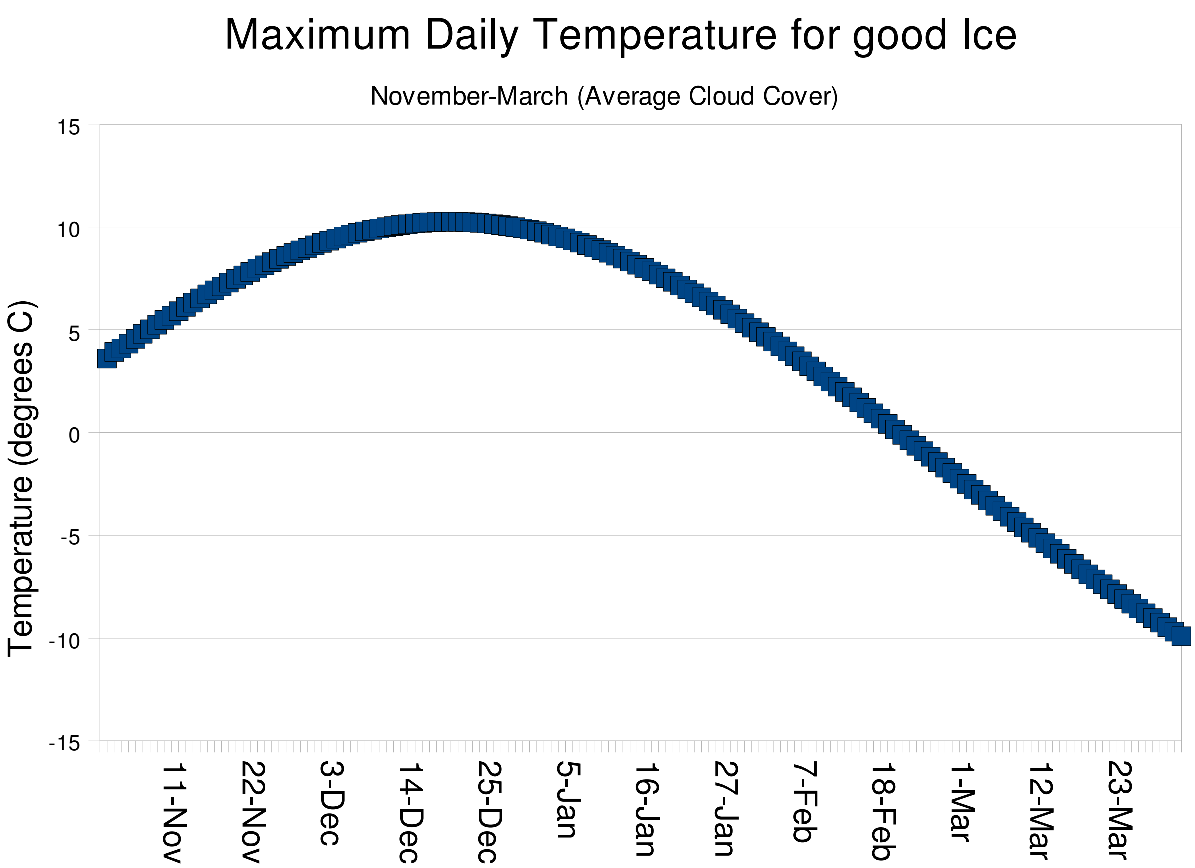
Alternatively, the results expressed in relative compressor load based on running time:



Alternate Output: Maximum hard ice Temperatures

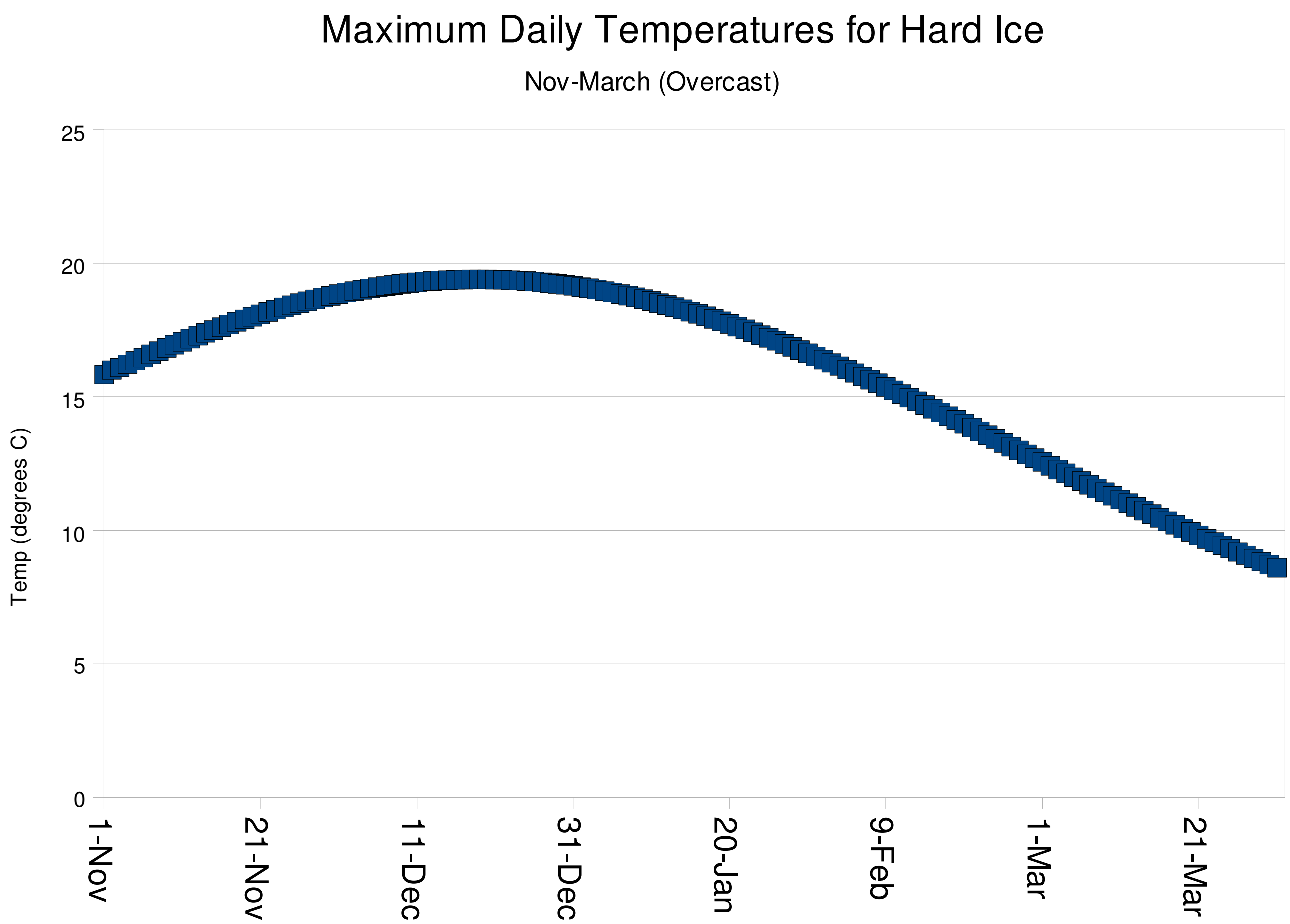
The system can also be solved for $T_{air\,max}$ across the time period to give the maximum air temperature at which an outdoor compressor cooled rink can manage to keep hard ice. These results are of particular use in the determination of the ideal outdoor ice season.

When viewed across the entire time period of November 1st to March 31st, at the solar maximum of any given day (noon time), we see a maximum temperature on December 21st of +10 ∘ C and a climatologically improbable -10 ∘ C at the extreme spring end.



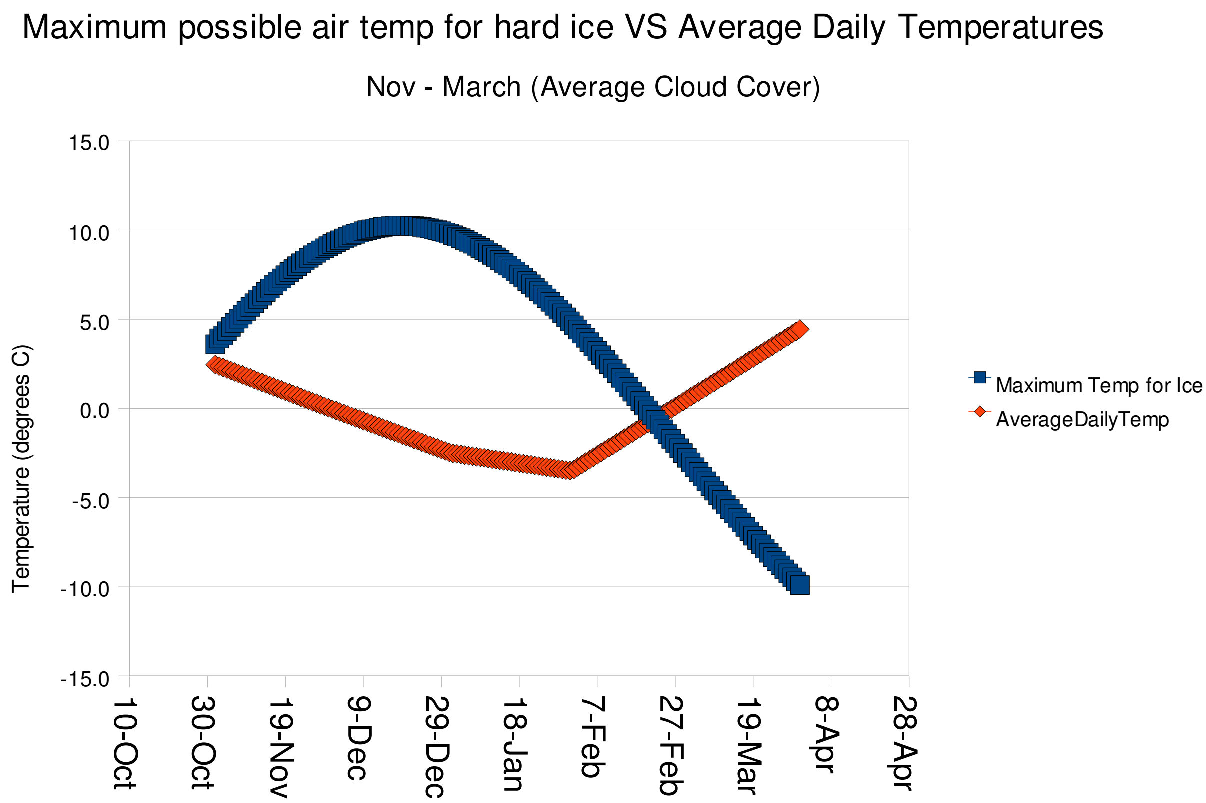
Note that ice remains solid at above freezing air temperatures during the months of November, December, January and most of February.

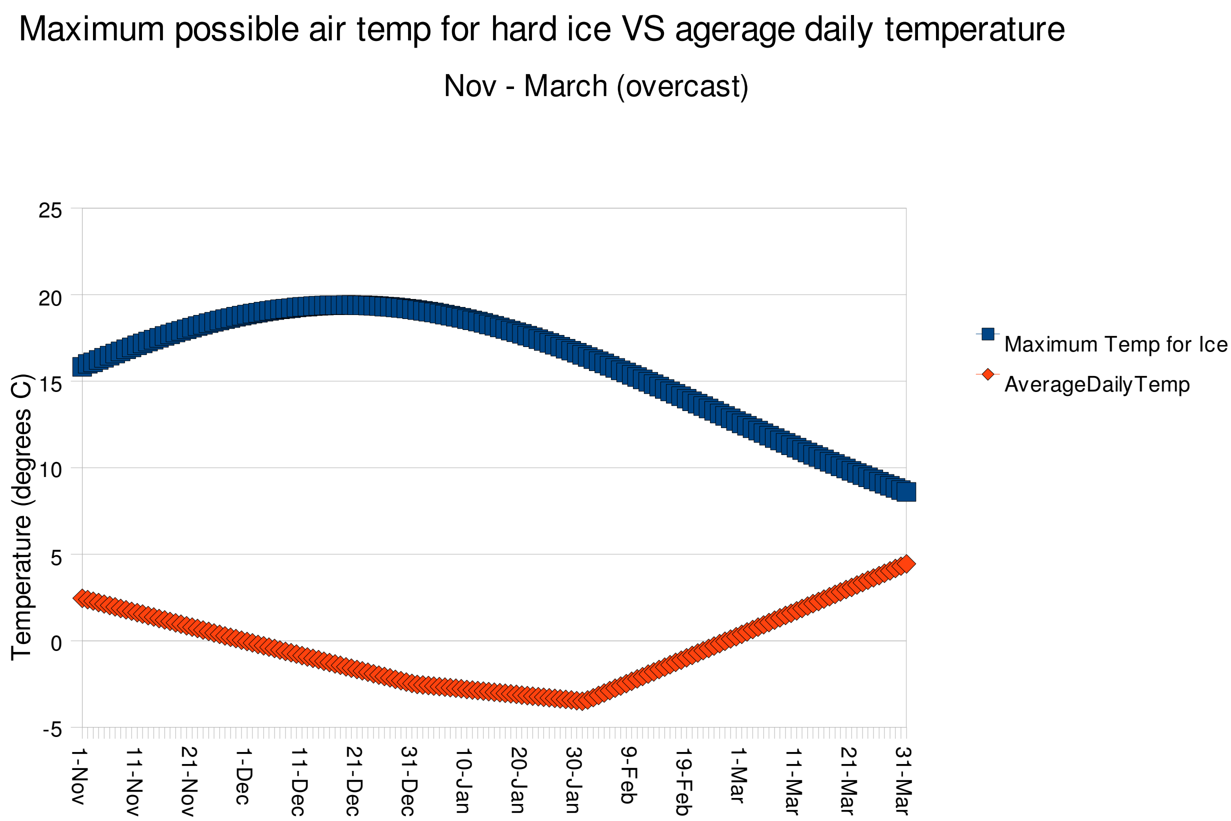
It can be seen by merely increasing cloud attenuation (simulating overcast conditions), the maximum temperature at which the ice can be kept solid is drastically increased over the entire season.



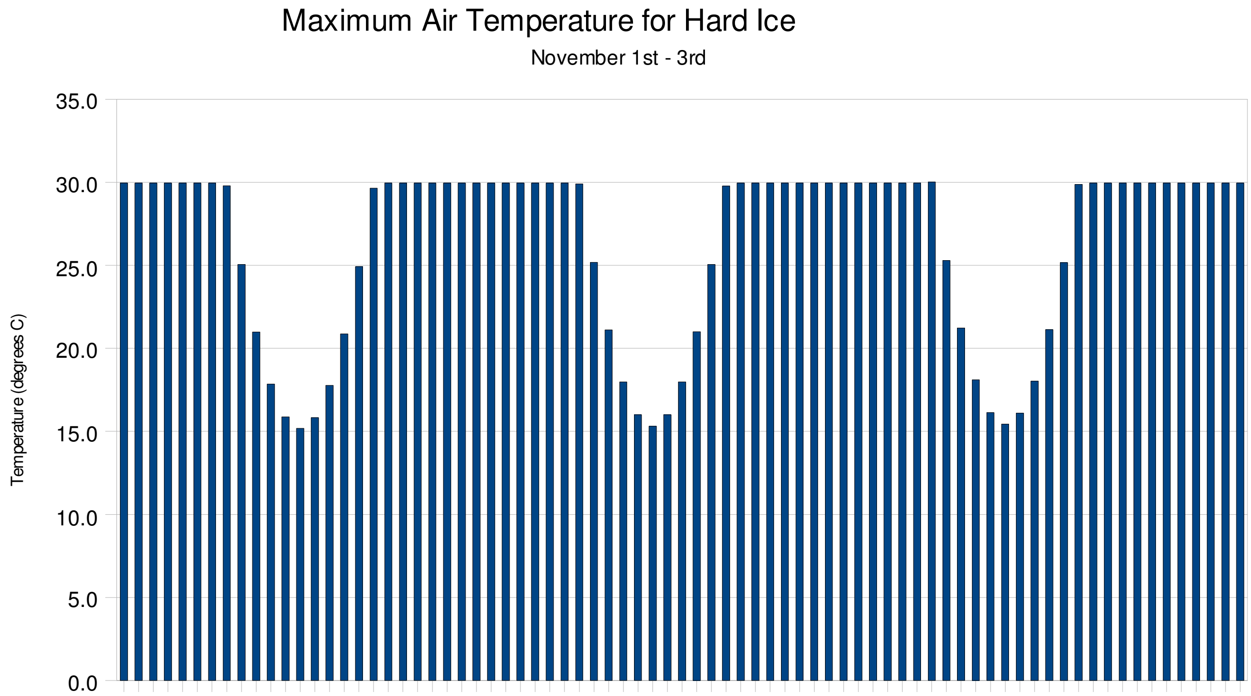
The above figure is very illuminating and fits with our empirical observations that outdoor ice rinks can hold ice at almost 20 ∘ C in overcast skies around the winter solstice. The time interval on which ice can be held at up to 15 ∘ C extends from the 1st of November to February 9th.

Now we consider the above scenarios when displayed against the historical average daily temperatures:





There is interesting information to be gathered at a much higher temporal resolution (values are calculated for every hour across the entire time interval). For instance, the hourly picture for the first few days of November (fair skies) shows the strong influent of the solar angle on temperature thresholds for the ice. Maximum allowable air temperature fluctuates 15°C over the course of one day.



Maximum Air Temperature for Hard Ice
November 1st - 3rd
Temperature (degrees C)
35.0
30.0
25.0
20.0
15.0
10.0
5.0
0.0